

# BRODA 社の SOBOL 擬似乱数ジェネレーターによる正確でハイパフォーマンスなシミュレーション

この記事は、Intel Tech.Decoded で公開されている「[Toward Accurate and Highly Performant Simulations with BRODA® SOBOL Quasi-random Number Generator](#)」の日本語参考訳です。

擬似乱数ジェネレーターは、さまざまな AI アルゴリズム (例: 粒子群の最適化) や、金融やリスク管理でよく使用される擬似モンテカルロ・シミュレーションにおいて、不一致度の低い乱数列のソースとして広く使用されています。BRODA 社が開発した SobolSeq65536 ジェネレーターが生成する Sobol' 数列は、数値積分の収束性を高める一様性条件 (特性 A と特性 A') を満たしており、既存の擬似乱数ジェネレーターよりも速度と精度の両方において優れています [2]。

インテル® oneAPI マス・カーネル・ライブラリー (インテル® oneMKL) は、インテルベースのシステムで最も高速で、最も多く使用されている数学ライブラリーです。演算処理ルーチンを高速化し、アプリケーションのパフォーマンスを向上させ、開発期間を短縮します [5]。SobolSeq65536\_MKL は、SobolSeq65536 数列ジェネレーターにインテル® oneMKL が組み込まれたものです。SobolSeq65536\_MKL は、同じ統計的性質になるように SobolSeq65536 の方向比を使用しています。インテル® oneMKL は、インテル・ハードウェア上で大規模シミュレーションに不可欠な BRODA の SOBOL の数列生成パフォーマンスを大幅に向上させるとともに、インテル® Xe グラフィックス・ハードウェア上でインテル® oneMKL DPC++ API を用いて SobolSeq65536 ジェネレーターを実行できるようにしました。

## SobolSeq65536 と SobolSeq65536\_MKL ジェネレーターの統計的性質

SobolSeq65536 ジェネレーターによって生成される不一致度の低い Sobol' 数列は、既存のジェネレーターの出力数列よりも統計的性質が優れています [2]。有名な金融ベンチマークの「ヨーロッパ型オプションの価格付け」と「アジア型オプションの価格付け」について、S. Joe 氏と F. Y. Kuo [3] (Joe & Kuo) 氏の方向比で BRODA の SobolSeq65536 ジェネレーターと Sobol' 数列ジェネレーターを比較した結果を以下に示します。

次のパラメーターでヨーロッパ型コールオプションの価格付けについて考えます:  $S_0 = 100$ ,  $K = 100$ ,  $r = 0.0$ ,  $\sigma = 0.2$ ,  $T = 1y$ 。アット・ザ・マネー (ATM) コールの理論的なブラック-ショールズ価格は 7.965667455 です。標準的なアルゴリズムでは、オプションの価格は次の  $d$  次元の積分として表されます。

$$C = e^{-rT} \int_{H^d} \max[0, (S_0 \exp[(r - \frac{\sigma^2}{2})T + \sigma \sqrt{\frac{T}{d}} \sum_{j=1}^d \Phi^{-1}(u_j)] - K)] du_1 \dots du_d.$$

「アジア型オプションの価格付け」では、次のパラメーターで幾何平均アジア型コールオプションの価格付けについて考えます:  $S_0 = 100$ ,  $K = 100$ ,  $r = 0.05$ ,  $\sigma = 0.2$ ,  $T = 0.5y$ 。標準的なアルゴリズムでは、幾何平均アジア型コールオプションの価格は次の  $d$  次元の積分として表されます。

$$C = e^{-rT} \int_{H^d} \max[0, (\left[ \prod_{i=1}^d S_0 \exp[(r - \frac{\sigma^2}{2})t_i + \sigma \sqrt{\frac{T}{d}} \sum_{j=1}^i \Phi^{-1}(u_j)] \right]^{1/d} - K)] du_1 \dots du_d.$$

「ヨーロッパ型オプションの価格付け」と「アジア型オプションの価格付け」のベンチマークにおける、2乗平均平方根誤差 (RMSE) の積分結果 (20 回の実行の平均) を以下に示します (次元  $d = 1024$ )。

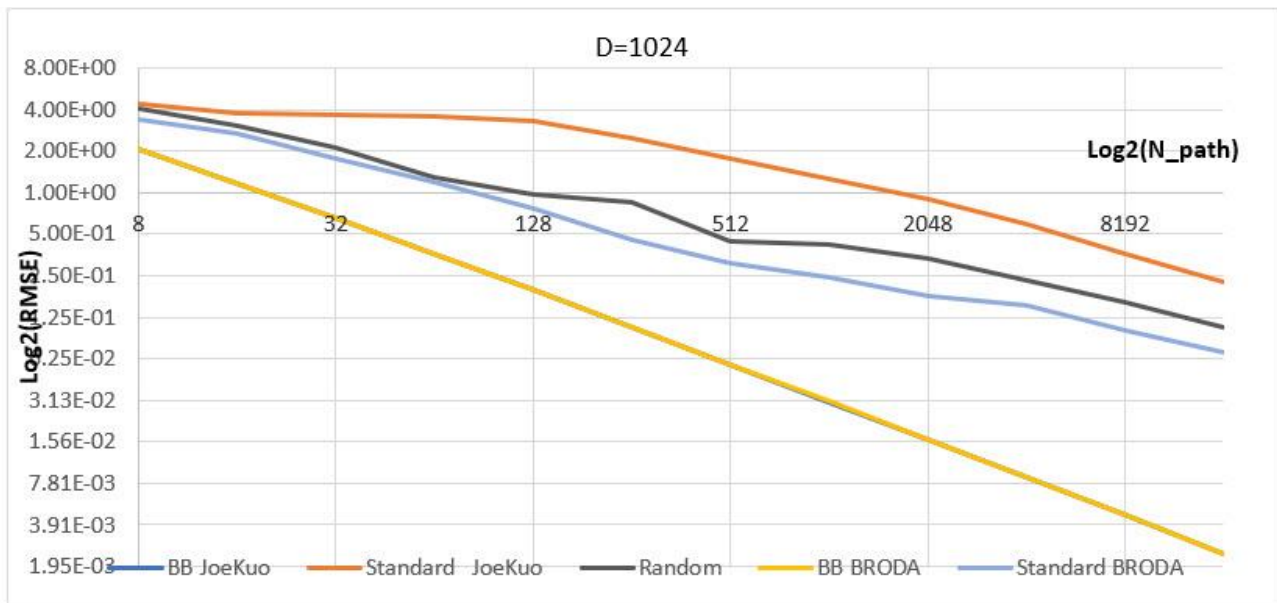


図 1. ATM コールの RMSE 積分

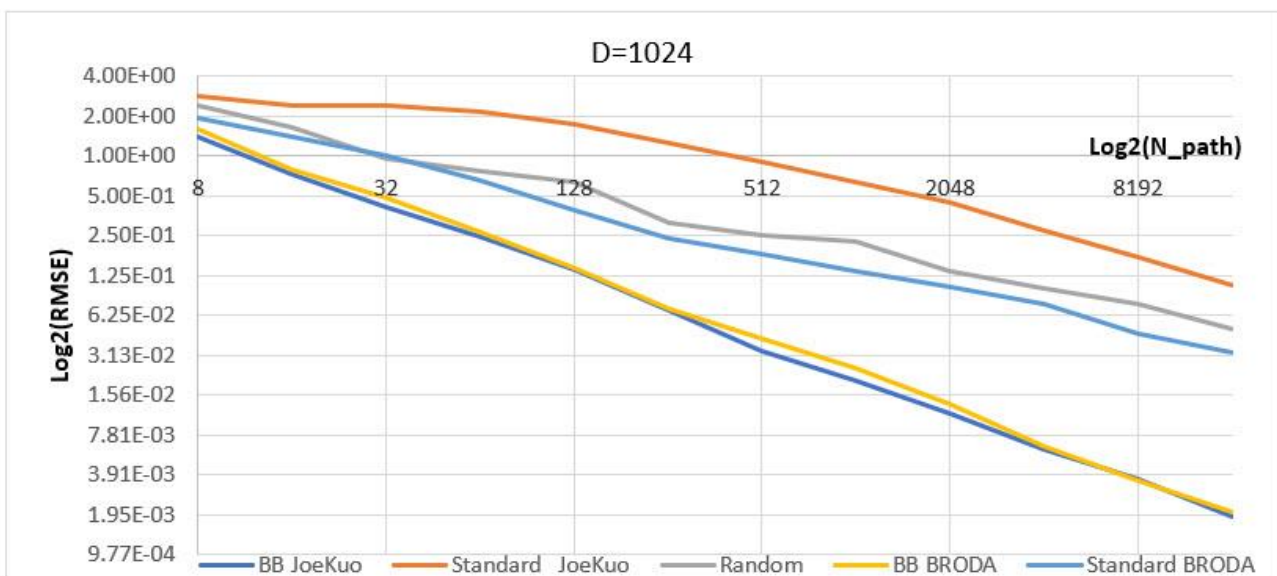


図 2. アジア型コールの RMSE 積分

BRODA と Joe & Kuo のジェネレーターを比較すると、Standard 方式の場合 Joe & Kuo は BRODA よりもはるかに効率が悪く、乱数ジェネレーター (MC 方式) よりも効率が悪いことが分かります。Brownian bridge (BB) の場合、どちらのジェネレーターも同様のパフォーマンスを示します。

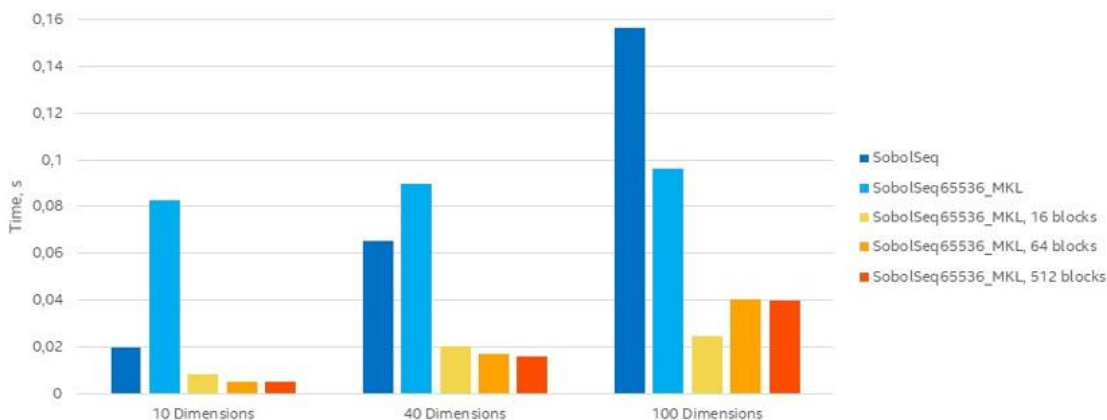
## インテル® oneMKL によるパフォーマンスの高速化

インテル® oneMKL のベクトル統計コンポーネントは、Sobol' 擬似乱数ジェネレーターの実装 (インテルの CPU と X® グラフィックスをサポート) を含む、一般的な乱数ジェネレーターのセットを提供します。Sobol' 擬

似乱数ジェネレーターの実装も、初期化時にユーザー定義のパラメーターの登録を受け付けます。これに基づいて、インテル® oneMKL を BRODA の SobolSeq65536 ジェネレーターに統合しました。

SobolSeq65536\_MKL (インテル® oneMKL が統合された SobolSeq65536) には、ポイントごとにアクセスする「シングルポイント」バージョンと 1 回の関数呼び出しでいくつかの Sobol' ベクトルを取得できる「ブロック」バージョンがあります。

SobolSeq65536 と SobolSeq65536\_MKL で  $2^{20}$  個の疑似乱数を生成したパフォーマンス (10 回の実行の平均) を図 3 に示します。

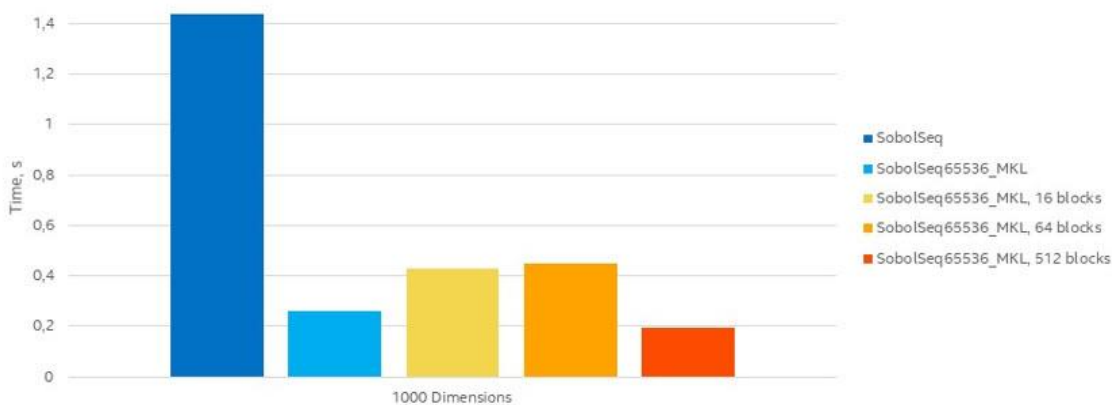


The benchmark results reported above may need to be revised as additional testing is conducted. The results depend on the specific platform configurations and workloads utilized in the testing, and may not be applicable to any particular users' components, computer system or workloads. The results are not necessarily representative of other benchmarks and other benchmark results may show greater or lesser impact from mitigations. Performance results are based on testing as of 04/23/2020 and may not reflect all publicly available security updates. See configuration disclosure for details. No product can be absolutely secure.

Software and workloads used in performance tests may have been optimized for performance only on Intel microprocessors. Performance tests, such as SYSmark and MobileMark, are measured using specific computer systems, components, software, operations and functions. Any change to any of those factors may cause the results to vary. You should consult other information and performance tests to assist you in fully evaluating your contemplated purchases, including the performance of that product when combined with other products. For more complete information visit [www.intel.com/benchmarks](http://www.intel.com/benchmarks).

Configuration: Testing by Intel as of 04/23/2020. Intel® Xeon® Platinum 8280L 2x28@2.7GHz using Intel® Math Kernel Library 2020 U2. Benchmark Source: BRODA.

Optimization Notice: Intel's compilers may or may not optimize to the same degree for non-Intel microprocessors for optimizations that are not unique to Intel microprocessors. These optimizations include SSE2, SSE3, and SSE4.2 instruction sets and other optimizations. Intel does not guarantee the availability, functionality, or effectiveness of any optimization on microprocessors not manufactured by Intel. Microprocessor-dependent optimizations in this product are intended for use with Intel microprocessors. Certain optimizations not specific to Intel microarchitecture are reserved for Intel microprocessors. Please refer to the applicable product User and Reference Guides for more information regarding the specific instruction sets covered by this notice. [Notice revision #20110804](#)

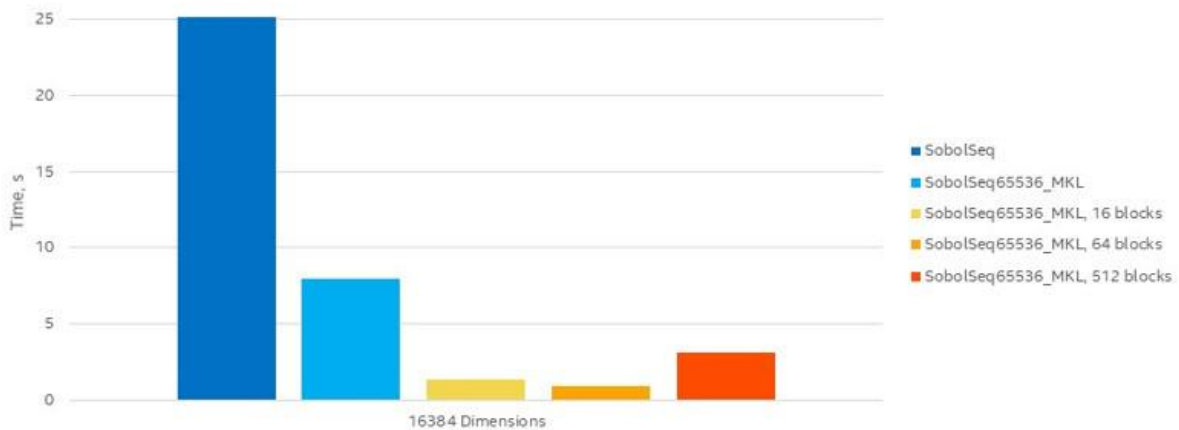


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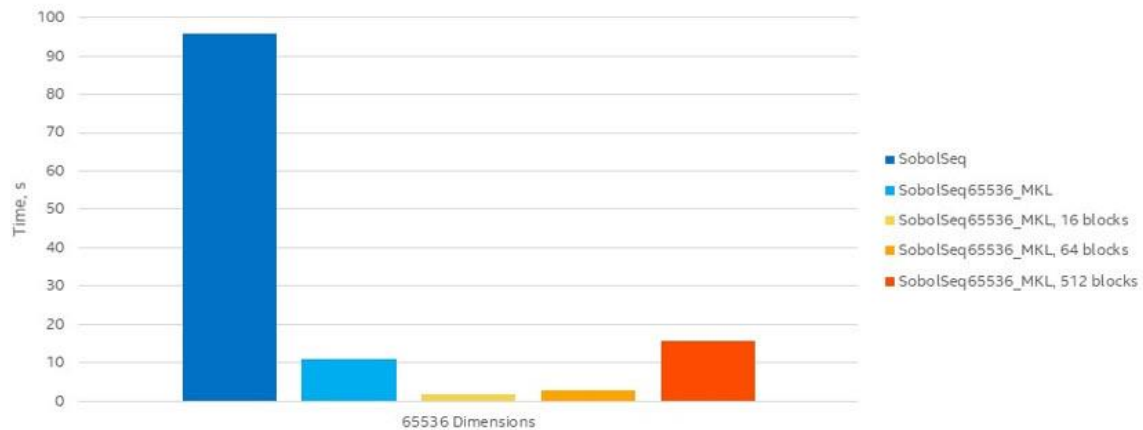


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図 3. SobolSeq65536 と SobolSeq65536\_MKL のパフォーマンス比較

グラフから、SobolSeq65536\_MKL の「ブロック」バージョンは SobolSeq65536 のパフォーマンスを大きく上回っていることがわかります。SobolSeq65536\_MKL は SobolSeq65536 と比較して、10 次元で 4 倍、100 次元で 4 倍、1000 次元で 7.9 倍、16384 次元で 27.7 倍、65535 次元で 52.7 倍のスピードアップを達成しました。

## まとめ

BRODA の SobolSeq65536 ジェネレーターは不一致度の低い 65536 次元の Sobol' 数列ジェネレーターで、速度と数列の統計的性質の両方において既存のジェネレーターを上回っています。SobolSeq65536\_MKL は、SobolSeq65536 数列ジェネレーターに Intel® oneMKL が組み込まれたものです。SobolSeq65536\_MKL ジェネレーターの「ブロック」バージョンは、Intel® ハードウェアのベクトル化機能を利用できるため、「シングルポイント」バージョンと比較して大幅なパフォーマンス向上が見られました。

## 参考資料

- [1] BRODA Ltd. <http://www.broda.co.uk> (2020).
- [2] Sobol' I., Asotsky D., Kreinin A., Kucherenko S. Construction and Comparison of High-Dimensional Sobol' Generators, *Wilmott*, Nov:64-79, 2011, [http://www.broda.co.uk/doc/HD\\_SobolGenerator.pdf](http://www.broda.co.uk/doc/HD_SobolGenerator.pdf)
- [3] Joe S., Kuo FY. Constructing Sobol sequences with better two-dimensional projections. *SIAM Journal on Scientific Computing*. 2008;30(5):2635-54.
- [4] Bianchetti M., Kucherenko S., Scoleri S., Pricing and Risk Management with High-Dimensional Quasi Monte Carlo and Global Sensitivity Analysis, *Wilmott*, July, pp. 46-70, 2015, [http://www.broda.co.uk/doc/PricingRiskManagement\\_Sobol.pdf](http://www.broda.co.uk/doc/PricingRiskManagement_Sobol.pdf)
- [5] Intel® oneAPI Math Kernel Library – <https://software.intel.com/content/www/us/en/develop/tools/math-kernel-library.html>
- [6] Intel® oneAPI Math Kernel Library Vector Statistics – Sobol quasi-random number generator. <https://software.intel.com/content/www/us/en/develop/documentation/mkl-vsnotes/top/testing-of-basic-random-number-generators/basic-random-generator-properties-and-testing-results/sobol.html>

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## 製品とパフォーマンス情報

<sup>1</sup> 実際の性能は利用法、構成、その他の要因によって異なります。詳細は、[www.intel.com/PerformanceIndex](http://www.intel.com/PerformanceIndex) (英語) を参照してください。